

1. (*Exeter 1.92.3*) Most positive integers can be expressed as a sum of two or more consecutive positive integers. For example,

$$24 = 7 + 8 + 9$$

$$36 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$$

$$51 = 25 + 26$$

A positive integer that cannot be expressed as a sum of two or more consecutive positive integers is therefore *interesting*. The simplest example of an interesting number is 1.

- (a) Find three more interesting numbers.
(*Hint: If you're not sure where to start, test the first 10 positive integers.*)
- (b) Show that 14 is not an interesting number.
- (c) Show that 82 is not an interesting number.
- (d) Can you find any interesting numbers that are odd?
- (e) What happens when you add two consecutive positive integers?
- (f) What happens when you add three consecutive positive integers?
- (g) What happens when you add four consecutive positive integers?
- (h) What happens when you add five consecutive positive integers?
- (i) What happens when you add n consecutive positive integers?
- (j) Find three ways to show that 190 is not an interesting number.
- (k) Find three ways to show that 2004 is not an interesting number.
- (l) How many interesting numbers precede 2018?